Closing Fri: 2.7, 2.7-8
Closing Tues: 2.8
Closing next Thurs: 3.1-2
Closing next Fri: 3.3 (last before Exam1)

2.7-8 Derivatives

Recall: We defined the slope of the tangent line to $f(x)$ at $x = a$ by

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

We call this value the **derivative** and denote it by $f'(a)$.

(From HW 2.7-8/6)

Entry Task: An object is moving on a straight line and its position is given by $p(t) = \frac{t^2}{t-1}$ feet.

Find $p'(3)$.

Notes/Observations:

1. We call $f'(a)$ the **derivative** of $f(x)$ at $x = a$.

2. Graphically, $f'(a)$ is the **slope of the tangent line** to $y = f(x)$ at $x = a$.

3. This is equivalent to saying $f'(a)$ is the **instantaneous rate of change** for $y = f(x)$ at $x = a$.

4. Given $y = f(x)$.
   Units of $f'(a)$ are $\frac{y\text{−}units}{x\text{−}units}$.
   For example,
   if x = hours and $y = f(x)$ = miles,
   then $f'(x)$ = miles/hour.

5. The **tangent line** to $y = f(x)$ at $x = a$ will always look like

$$y = f'(a)(x - a) + f(a)$$

(Like HW 2.7/6)

Example: Consider the ellipse

$$x^2 + 2y^2 = 6$$

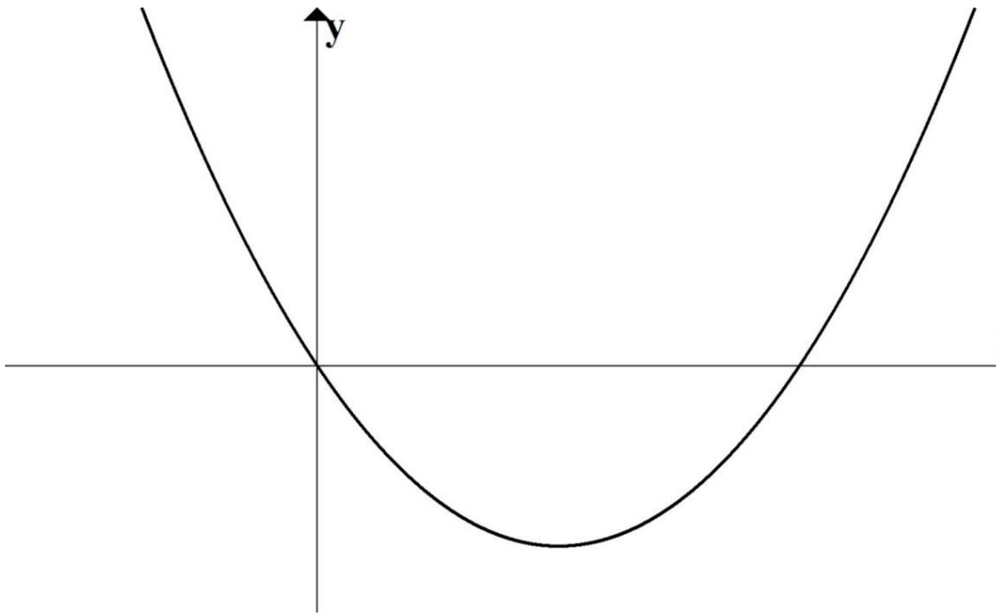
Find the slope of the tangent line at

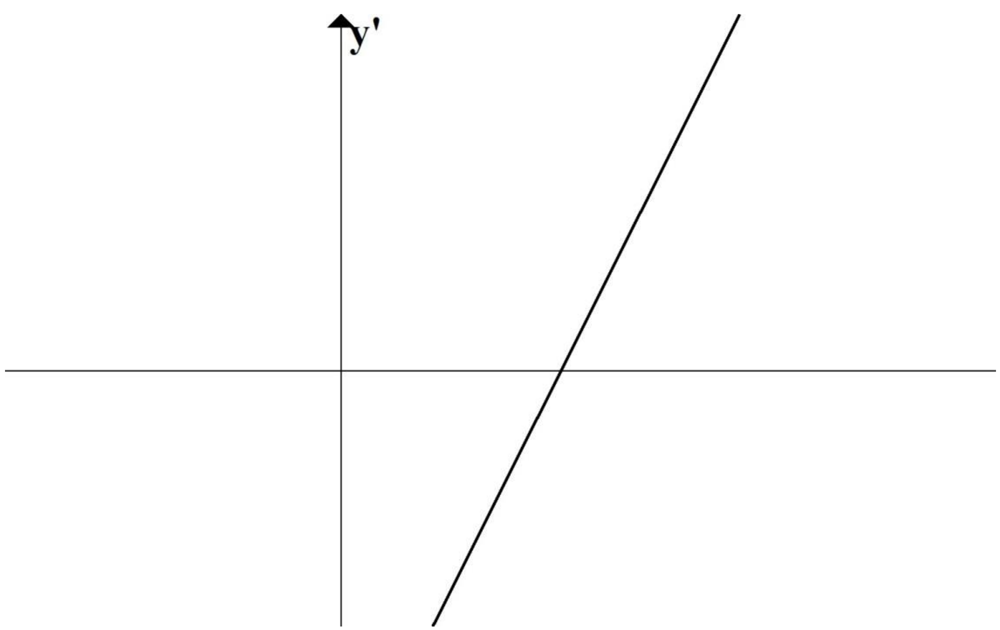
$$(x, y) \;=\; (\sqrt{2}, 1)$$

2.8 The derivative function

Example: Let $f(x) = 2x^2 - 3x$

1. Find $f'(3)$.
2. Find $f'(x)$.





Notes/Observations:

Given $y = f(x)$.

- $y = f'(x)$ is a new function.
- $f(x) =$ "height of the graph at *x*"
- $f'(x) =$ "slope of $f(x)$ at x"
- Again, $f'(x)$ is the "instantaneous rate of change" (speedometer speed)
- The units of $f'(x)$ are $\frac{\text{y−units}}{\text{x−units}}$.

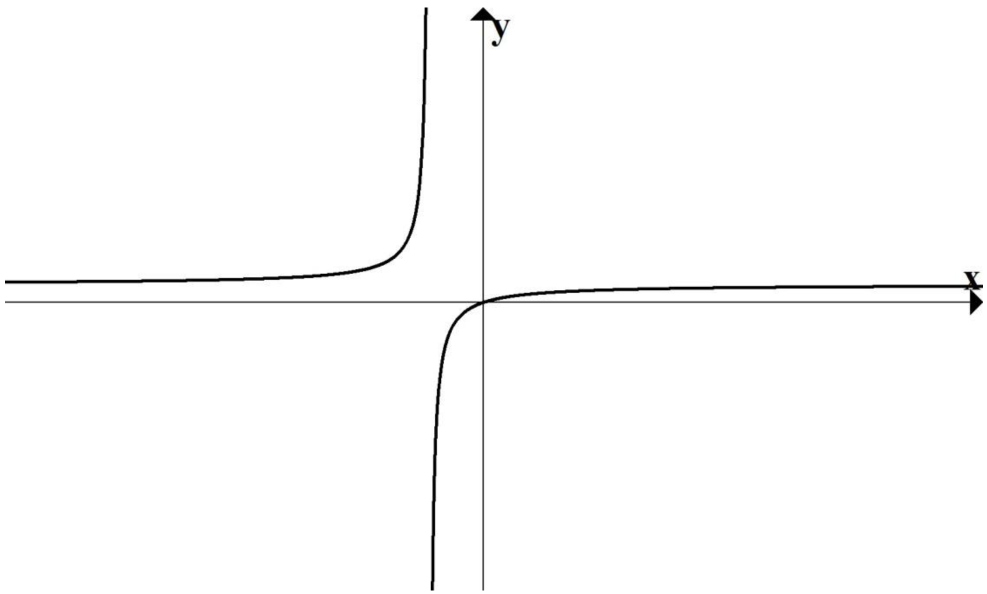
Fundamental to all applications:

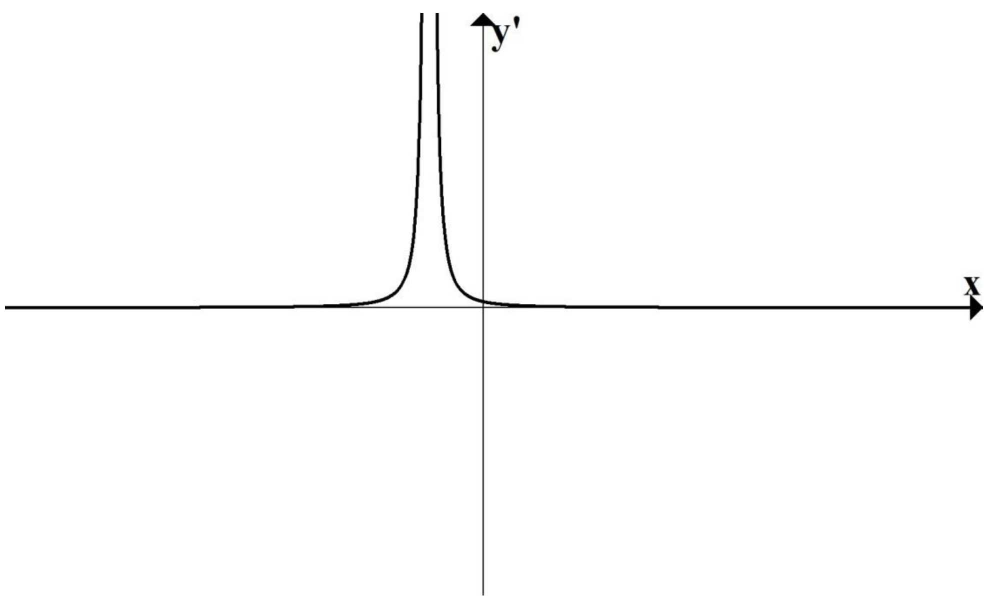
$y = f(x)$	$y = f'(x)$
horiz. tangent	zero
increasing	positive
decreasing	negative

Example:

$$g(x) = \frac{2x}{x+3}$$

1. Find $g'(2)$.
2. Find $g'(x)$.


y
x


y'
x

Notation:

Early we found

if $f(x) = 2x^2 - 3x,$

then $f'(x) = 4x - 3.$

Other ways to write this include:

$$y' = 4x - 3$$

$$\frac{dy}{dx} = 4x - 3$$

$$\frac{d}{dx}(2x^2 - 3x) = 4x - 3.$$

Later we will also discuss:

$$f''(x) = y'' = \frac{d(dy/dx)}{dx} = \frac{d^2y}{dx^2}$$

Example:

if $y = f(x) = 2x^2 - 3x,$

then $y' = f'(x) = 4x - 3$

and $y'' = f''(x) = 4$

which can also be written as

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(4x - 3) = 4$$

Differentiability

Sometimes we can have a place where "slope of tangent" doesn't make sense.

Definition:

We say a function, $y = f(x)$ is **differentiable** at $x = a$ if the following limit exists:

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Otherwise it is not differentiable at $x = a$.

In order to get differentiable:

1. It must be defined at x = a.
2. It must be continuous at x = a.
3. The "slope" must be the same from both sides.

Examples:

1. $f(x) = \frac{1}{x-3}$

$f(x)$ is not defined at $x = 3.$

Thus, it is not continuous at $x = 3.$

And it is not differentiable at $x = 3.$

2. $g(x) = \begin{cases} 2x - 1 & , \text{if } x < 2; \\ x^2 & , \text{if } x \geq 2. \end{cases}$

$g(x)$ is defined at $x = 2.(g(2) = 4)$

But, g(x) is not cont. at $x = 2$

because $\lim_{x \to 2^-} g(x) = 3$ and

$\lim_{x \to 2^+} g(x) = 4.$

Thus, g(x) is not differentiable at x=2.

3. $k(x) = |x|$

$k(x)$ is defined at $x = 0$. (k(0) = 0)

$k(x)$ is continuous at $x = 0$.

But k(x) is not differentiable at x = 0.

(The slope from the left is -1 and

the slope from the right is +1)

There is a "sharp point" at x = 0.

4. $j(x) = x^{1/3}$

$j(x)$ is defined at $x = 0$. (*j*(0) = 0)

$j(x)$ is continuous at $x = 0$.

But j(x) is not differentiable at x = 0.

(The slope goes to infinity as you get

close to 0).

There is a vertical tangent at x=0.